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POLISH ACADEMY OF SCIENCES

Institute of Basic Technical Problems

Department of Vibrations

The Summaries of the Selected Papers of the Mechanical Vibrations Laboratory during the years 1956-1960

Technical
Editorial Secretary
R. Miklauszewski

Vibrations

of Non-Linear Discrete Systems

1:

Ziembe, S. On the certain properties of systems with the strong non-linearity. (In Polish).
O pewnych własnościach układow o silneh nieliniowości.
Biul.Wojsk.Akad Techn. No 30/47/, 1955.

In this paper the author discusses the typical properties of the linear systems with one degree of freedom. The four cases of free and forced vibrations both damped and undamped are described. In the section of non-linear systems there are analyzed the equations of phase trajectories, the method of isoclines and the "sewing" method.

The last part of the paper deals with quasi-linear systems and nonlinear vibrations with "small amplitude".

2:

Ziemba, S. The influence of mass and internal friction on free torsional vibrations of a bar. (In Polish).
Wpływ masy własneh pręta i tarcia wewnę trznego na swobodne drgania skrętne. Arch.Mech.Stos. IX, 1/157.

In introduction there are reminded the basic differential equations of torsional vibrations of bar. The author gives the formulae for the period of vibrations and mentions that torsional vibrations of cylindrical bar are used for determining the modulus of elasticity and for purposes of measuring or comparing the damping properties of various materials.

Then the differential equation of torsional vibrations of the bar with variable cross-sections is deduced. Internal friction and the mass of the bar is taken accurately into consideration. The torsional vibrations are treated as vibrations of an infinite number of degrees of freedom. In the second part of the paper are considered the free vibrations of cylindrical shaft with internal and external damping. There are discussed the various cases of different boundary conditions: with free ends (without masses attached and without forces) and with one fixed and the other remaining free.

3:
Skowronski, J. M. Linear damping of a single impulse by a non-linear shock absorbing system. (In Polish). Liniowe tłumienie nieliniowch amortyzacji pojedyńczego impulsu. Rozprawy Inż. Vol. X. CII. 1957, 1. See also Zentralblatt 84 Band. Heft 1, S. 201.

After discussing the design problems of the shock-absorbing system considered, and also physical assumptions and the scope of investigation, a model of the system is established with the corresponding equation of motion. The existing results of the qualitative analysis of case under consideration are summarized. Then, the author passes to a description of approximate methods of quantitative analysis for each particular range of damping parameters and elastic characteristic of the rigid type. The calculations are accompanied by numerical examples. On the basis of approximate equations of phase trajectories, the discussion is entered into the character of damping and the alternating motion of the shock-absorbing system depending on various damping parameters and elastics characteristic. A numerical example of a motion like that for various values of these parameters is included. Attention is drawn to the necessity of an analysis of that kind for the designer of the shock-absorbing system in every case of design. This is necessary for selecting the optimum values of parameters.

4:

Ziemba, S. The influence of deafing coverings on the vibrations. (In Polish).
Wpływ pokryć głuszących na drgania.
Biul.Wohsk.Akad.Techn.Vol.VI Mp 2 (73) 1957.
The author considers the scheme of the collaboration of damping coverings and elements are being covered. Some data for the choosing the place and for the manner of using the damping coverings are defined. The directive results for the proofs and for the evaluation of damping properties of different kinds of pastes had been obtained.

5:

Ziemba, S. The graphical methods of plotting of phase trajectories of the non-linear free vibrations of mechanical systems with one degree of freedom. (In Polish).
Wykreślne metody wyznaczania trajektoril fzowych nieliniowysh drgań swobodnych układów mechanicznych o jednym stopniu swobody. Biul.WAT, Mp 3. 1957.

The author exhibits the method of plotting the phase trajectories of differential equations of form

$$\ddot{x} + h(x)\,g(\dot{x}) + f(x) = 0 \qquad (5)$$

Seven different cases are considered. The purpose of this paper is to illustrate the certain succesive way of finding the graphical construction of phase trajectories for more complicated cases of equations (5).

6:

Ziemba, S. Dry friction vibration damping. (In English). O tłumieniu drgań przez tarcie suche. Arch.Mech.Stos. Vol. IX, 1957, z. 3. See Appl.Mech.Rev.Vol. XI No, 1958. Rev. 1985.

The paper deals with the proper treatment of the dry friction on the dependency of velocity of unconstant direction for the system of one degree of freedom. Particulary the free vibrations damped by dry friction are considered. Using the Lienard method of construction of the phase trajectories the differences in treatment of alternativity of resistance of dry friction are illustrated by these trajectories.

By the application of Lapunov's theorem on the investigations of stability of singular points of non-linear systems throughout the investigations of linearized system, the problems of existing, kinds and stability of singular points are discussed.

7:

Ziemba, S. The influence of viscosity damping on the form of the trajectories of free vibration. (In English). Wpływ tłumienia wiskotycznego na przebieg trajektorii drgań swobodnych. Arch.Mech.Stos.Vol.IX z.IV. 1957. See Appl. Mech.Rev. Vol. XII No 6, 1959m Rev. 2775.

The author introduces a new graphical construction of the phase trajectories for linear and non-linear damping. This new construction is more complicated than Lienard's method, but it shows better the influence of the damping on the behaviour of phase trajectories.

Using this constructions the author discusses the form of phase trajectories of vibrations with non-linear damping for different cases of the characteristics of damping. In certain cases there are shown the phase curves which may be determined effectively. Among them is contained the phase trajectory corresponding to the non-linear damping. This phase trajectory has the same form as the phase trajectories determined effectively.

8:

Ziemba, S. Free vibrations with damping of marked non-linear character. (In English).
Drgania swobodne przysilnie nieliniowym tłumieniu.
Arch.Mech.Stos.Vol.IX, s. 5, 1957.

In connection with linear damping of systems with one degree of freedom of linear elastic characteristic and with "critical damping" two kinds of non-linear damping "hard" and "soft" are considered.

There are presented the conclusions of Sansone theorem for hard damping. A behaviour of the system with combined linear damping and non-linear soft damping is studied. There is shown that system of these damping vibrations can perform the finite number of alternative displacements and then the system tends asymptotically to equilibrium state in infinite time.

9:

Ziemba, S. A list of the problems of mechanical vibrations and dynamics of constructions. (In Polish).
Zestawienie problematyki z dziedziny drgań mechanicznych i dynamiki konstrukcji. Zeszyty Probl.Nauki Pol.XI, 1957.

The well-known notions as dynamical factor, safety factor and permissible stresses should be revised. It is caused by the fact that almost every element or construction is the system which can perform the vibrating motion and also the characteristics of materials are non-linear. The investigations of physical properties of solids and the fatigue strength should be continued.

10:

Skowroński, J. M. Ziemba, S. Application of delta method to the investigations in a strong non-linear vibrating mechanical system with several degrees of freedom. (In Polish).
Zastosowanie metody delta do badania silnie nieliniowych mechanicznych układów drgających o kilku stopniach swobody. Bull.WAR 1, 1958.

The purpose of this paper is the application of delta method to the investigation of the vibrations of the strong non-linear mechanical systems with n degrees of freedom, both autonomous and nonautonomous and with arbitrary kinds of coupling. The first part deals with the systems of several degrees of freedom. For the engineer engaged in this type of vibrating motion, the delta method may be useful and practical to the quantitative and qualitative analysis. For certain kinds of motion of the strong non-linear systems with several degrees of freedom, it is, perhaps for present time, the only way of investigation of conduct of phase trajectories. Several numerical examples of the application of this method and certain ways of drawing the phase trajectories and examples of finite equations vibrating motion are given. Then the main purpose of this paper is to popularize the known delta method.

11:

Ziemba, S. On certain method of investigations of non-elasticity of solids. (In Polish).
Opewej metodzie badania niesprężystości ciał stałych.
Buil.WAT No. 15, 1958.

It is an attempt of experimentally estimating the characteristics of internal friction of the solids in an assumption that elastic properties are known. The experiment contains the investigation of steady periodic vibrations caused by known external periodic force.

12:

Ziemba, S. Free vibration of systems of one degree of freedom with non-linear elastic characteristic and non-linear viscous type damping. (In Polish).

Drgania swobodne ukladow o jednym stopniu swobody, o nieliniowej charakterystyce sprezystej i nieliniowym wiskotycznym tlumieniu. Arch.Mech.Stos. Vol. X, z 2, 1958.

The case considered in this paper is a further development of the cases treated previously by G. Sansone, R. Gutowski and the present author and contains a certain generalization of the problem of non-linear free vibration of systems of one degree of freedom.

13:

Skowroński, J. M. A question concerning the non-linear (soft) spring-characteristic in the free-damping vibrations of elastic panel. (In Polish).
Pewne zagadnienie nieliniowej "miękkiej charakterystyki sprężystej w przypadku swobodnych drgań tłumionych o skończonych amplitudach wiotkiej posłoki cylindrycznej. Czasopismo techniczne (of Cracow Technical University) Vol. 63, 1958, 3(9).

The problem of solution of

$$m\ddot{x} + l\dot{x} + F(x) = 0$$

is regarded theoretically, with F (x) uncontinuous, composed of three different strong non-linear (soft) forms. The uncontinuous point of F (x) is corresponding to the free jump phenomenon of the elastic-spring panel. The forms of the shape of phase plane trajectories are regarded, especially in the region of this incontinuity of $F(x)$ according to a jump point. The author obtains several concrete conclusions about the qualitative analysis of the system (of phase trajectories image) using several special methods. The damping is assumed linear. There is given the exact discussion of the spring characteristic parameters towards the constructions-synthesis of the system.

14:

Skowroński, J. M., Ziemba, S. <u>The problem of vibrations of non-autonomic systems with strong non-linearity.</u> (In English).

Zagadnienię drgań periodycznych nieautonomicznyeh układów silnie nieliniowych o dowolnym sprzężeniu.

Arch.Mech.Stos. Vol. X, z. 4, 1958.

See Math.Rev. Vol. 21, No. 4, Rev. 2383.

Zentralblatt 84 Band, Heft 1, s. 201.

For the system $\dot{x}_i = F_i(x_1, \ldots, x_{2s}, t)$ there is considered a homeomorphic analytical transformation T of Euclides phase space into itself. It has exhibited the existence of maximum region invariant in relation to this transformation and on the base of Brouwer theorem the existence of fixed point and thus the existence of periodic solutions. Some suggestions for investigations of stability of the solutions are given.

15:

Ziemba, S. Vibrations of mechanical systems with one degree of freedom and generalized forces not depending on an explicit manner in time. (In English).
Drgania układów mechanicznych o jednym stopniu swobody, w których siły uogólnione nie zależa wyraźnie od czasu. Arch.Mech.Stos. Vol. X, z. r, 1958.
See Appl.Mech.Rev. Vol. XII, No. 10, 1956, Rev. 4960.
Math.Rev.Vol. 20, No. 10, 1959, Rev. 6815.

In this paper the general case of non-linear free vibrations of system of one degree of freedom is considered. Elastic characteristic is non-linear and non-linear damping constitutes a function of both the velocity and the deflection. There is shown the way of investigation of vibrations by means of comparing their trajectories with trajectories of not damped non-linear vibration or almost linear vibration.

16:

Skowroński, J. M. A method of qualitative analysis of vibration discrete systems with strong non-linearity in the phase space. (In English).
Pewna metoda analizy jakościowej silnie nieliniowych, drgających układów dyskretnych w prezestrzeni fazowej. Arch.Mech.Stos.Vol. X, z 5, 1958.
See Zentralblatt 83 Band, Heft 1, x. 83.
Math.Rev.Vol. 21, No. 8, 1960, Rev. 5300.
Appl.Mech.Rev.Vol. 12, No. 11, 1959, Rev. 5487.

The qualitative character of the course of phase trajectories of vibrating motion of non-autonomous strong non-linear of n degrees of freedom system in 2n dimensional phase space can be determined by the discussion of hodograph of vector, on the base of tensor field of second valency, absolute derivative of this vector depended on the physical characteristics of the considered system. It is suggested the applications of this method to the qualitative synthesis of motion of such system.

17:
Skowroński, J. M.; Ziemba, S. Some complementary remarks on the delta method for determining phase trajectories of systems with strong non-linearity. (In English).
Pewne usupełnienia metody delta syznaczania trajektorii fazowych drgań układów silnie nieliniowych.
Arch.Mech.Stos.Vol.X, z 5, 1958.
See Zentralblatt 83 Band, Heft 1, s. 83.
Math.Rev.Vol. 20, No. 9, 1959, Rev. 6202.

A manner of the control of precision of the phase trajectories constructed by applying the delta method has been given. This criterion can be performed by using the parallel construction of curve obtained by raising the order of equation of motion. A certain possibility of increasing the exactness of construction by introducing the amendments is suggested.

18:

Ziemba, S. Simple cases of behaviour of bodies subjected to loads, taking internal friction into consideration. (In English).

Najprostsze przypadki zachowania się ciał sprężystych pod wpływem obciążen z uwzględnieniem tarcia wewnętrznego. Księga Jubileuszowa prof. dra inż. W. Wierzbickiego. W-wa 1959.

This is a report of theoretical investigations concerning vibration of visco-elastic bodies. It contains, beside the known solutions discussed as an example, also solutions published for the first time.

In the I and the II part free and forced vibration of a system with one degree of freedom is discussed among other problems, the vibrating mass being connected with the fixed point by means of a Maxwell or standard linear element. The III part deals with the propagation of waves in a viscoelastic meduim, the equations of motion being derived for a Voigt, Maxwell and standard linear body. The equations obtained constitute a basis for the considerations of the IV part devoted to free longitudinal vibration and free and forced flexural vibration of Voigt, Maxwell and standard linear bodies.

19:

Skowroński, J. M., Ziemba, S. The problem of boundedness of motion in certian mechanical systems. (In English). Sagadnienie ograniczoności ruchu pewnych układów mechanicznych. Proceedings of Vibration Problems No. 2, 1959, See Appl.Mech.Rev.Vol. 13, No. 8, 1960, Rev. 3919.

A model of structure and set of equations of its motion is considered. The conditions for tight hand sides of these equations originating from assumption of positive energy dissipation and characteristics of elasticity of system are determined.

On the basis of these conditions and the theorem of Demidovitch the conditions of boundedness of of vibrating motion are discussed. It was proved that for accepted physical assumptions the confined motion of system is bounded "in large" and among phase trajectories is the limit cycle determined by initial conditions i. e. all trajectories tend asymptotically to this limit cycle in the course of time $t \rightarrow \infty$.

20:

Skowroński, J. M. Some remarks on the phase space trajectories of the mechanical, non-autonomous, strong non-linear discrete systems. (In Polish). Pewne uwagi o trajektoriach w przestrzeni fazowej silnie nieliniowych mechanicznych, nieautonomicznych układów dyskretnych. Czasopismo Techniczne (of Cracow Technical University) Vol. 64, 1959, 2(18).

The general discrete dynamical system is regarded. with equations of the form

$$\ddot{q} + f_i(q, \dot{q}, t) = 0 \qquad i = 1, \dots n$$

with adequate initial conditions.

Where $q = q(q_1 \cdots q_n)$

$\dot{q} = \dot{q}(\dot{q}_1 \cdots \dot{q}_n)$ - the n-vector in the 2n-dimensional Euclidean Phase Space.

There are proposed some methods in the qualitative investigations (analysis and synthesis) of regarded systems. The author used the generalized "delta" method, which is taken in the 2n-phase space, as a qualitative method. This form of delta method was introduced and proved by the author in the previous papers. Several conclusions in relation to characteristics of system are given.

21:

Skowroński, J. M., Ziemba, S. Certain properties of mechanical models of structures. (In English). Pewne własności modelu mechanicznego ustroju sprężystego. Arch.Math.Rev.Vol. 21, No. 8, 1960, Rev. 5325.

A model of constructed system consisting of a set of discrete material points in three dimensional space, connected mutually with damper and spring with strong non-linear characteristics is considered. The system of points presents the distribution of mass of construction and connections corresponding to external and internal forces acting on the separate points of model. Standard boundary conditions of construction are replaced by corresponding conditions for coordinates of fixed points of model.

The equations of motion in Cartesian coordinates in three dimensional space of model and in configuration space are deduced. Potential of elastic forced and combined function of dissipative and conservative forces for fixed boundary condition of structure are described. The generalized forces are distributed into dissipative and conservative forces. Conditions of existence of generalized Lagrange's function and generalized dissipative function are given. It was proved that the character of dissipative forces determines why the system in a dynamical form in configuration space of generalized coordinates and in phase space are given; in phase space the previously introduced dissipation of the system is interpreted.

22:

Skowroński, J. M., Ziemba, S. <u>The boundedness of the motion of mechanical systems</u>. (In English). Orgraniczoność ruchu układów mechanicznysh. Proceedings of Vibrations Problems No. 3, 1959.

For the general case of discrete mechanical system with strong non-linear characteristics and arbitrary coupling for the case of positive energy dissipation in system and for bounded power of source of exciting forces it was proved that motion of system is bounded.

The proof is based on the theorem of Yoshizawa in which the condition of boundedness of solutions of equations is the existence of a function corresponding to Lapunov's function. It was proved that for the considered mechanical system such a function exists and its precise form is defined.

23:

Ziemba, S. Internal friction with micro-deformations taken into consideration. (In Polish).
Tarcie wewnętrzne przy uwzględnieniu mikroodkształceń plastycznych. Zag.Drgań Niel. No. 1, 1960.

The author considers the free vibrations in the system of one degree of freedom and the linear elasticity and damping characteristic depending on the velocity and the deflection

$$m\ddot{x} + kx + f(x)\,h(\dot{x}) = 0$$

A method for tracing the phase trajectories defined by this equation is given and the shape of the curves analysed.

The character of motion is shown to be influenced from the point of view of its alternativity by the damping characteristic and the initial conditions; the criteria of the alternativity of motion of a given system have been defined.

24:

Skowrondki, J. M. Free vibrations of a certain mechanical system with strong non-linear damping and with two degrees of freedom (In Polish).
Drgania Swobodne pewnego układu mechanicznego o silnie nieliniowym tłumieniu i dwóch stopniach swobody.
Zag. Drgań Niel. No 1 1960.

The author analyses the certain mechanical system with two degrees of freedom and strong non-linear characteristics. The motion of this system can be described by equations

$$\ddot{x}_1 + [l_1 + F_1(\dot{x}_1)]\dot{x}_1 - l_{12}\dot{x}_2 + k_1 x_1 - k_{12} x_2 = 0$$

$$\ddot{x}_2 + [l_2 + F_2(\dot{x}_2)]\dot{x}_2 - l_{21}\dot{x}_1 + k_2 x_2 - k_{21} x_1 = 0$$

with adequate initial conditions. G. Sansone, and S. Ziemba have investigated the motion of a similar system with one degree of freedom. It is interesting to generalize these considerations into the system regarded.

It is proved, that in the finite neighbourhood of the fundamental energy level $\left(\frac{dH}{dt} \leqslant 0\right)$ (H is the total energy of the system) the criteria of the oscillatory property of the motion of the considered system are the criteria of some corresponding linear comparative systems.

25:

Skowroński, J. M., On the possibility of the synthesis of some strong non-linear mechanical vibrating discrete systems. (In Polish).
O możliwościach syntezy pewnych silnie nieliniowych mechanicznych dyskretnych układów drgających.
Zag. Drgań Niel. No. 1. 1960.

Introducing the idea of synthesis of arbitrary mechanical system as the process for determining the type of characteristics of this system, which can be adequate to the arbitrary form of motion, the author suggests for realization of this process the simultaneously illustrative presentation of all discussed parameters of characteristics. The author uses the delta method which is adapted to this problem.

26:
Skowroński, J. M. The influence of damping on the character of the strong non-linear mechanical vibrations in the discrete systems. (In Polish).
Wpływ tłumienia na charakter drgań silnie nieliniowych mechanicznych układów dyskretnych. Sag. Drgań Niel. No. 2, 1960.

Almost every technically real construction system may be transformed to the discrete system, and especially to systems of the finite number of degrees of freedom. This provides the possibility of taking into consideration more fully the necessary physical and geometrical properties of the system concerning the Nonlinearity, sufficiently strong in general case of its characteristics. It is of great value because of the great quantity of the research difficulties. There are given the analysis and the investigation on the possibility of the synthesis and the influence of the positive damping on the character of motion (i.e. conditions of an aperiodicity and a periodicity of the vibrations), in a certain sufficiently generally modelled freely vibrating construction system with strongly non-linear characteristics and strongly non-linear arbitrary coupling. These systems are given as the discrete systems, described by the ordinary differential equations. The motion of these systems is considered in the sufficiently large, arbitrary, open and limited surrounding of the point of the steady state equilibrium position. This paper contains six sections in which are described the problems under consideration. Sec. 1 contains an introduction to the whole set of problems under consideration and gives the examples of the technical applications. In Sec. 2 there is applied the adequate generalized model of regarded mechanical systems, as a "chain" of descrete masses coupled "one to one" with strong non-linear dampers and springs. The equations of motion for this system are introduced. For so modelled system in Sec. 3 there are discussed the physical conditions of damping and spring characteristics, i.e. if the right sides of equations and furthermore several conceptions into theory of this characteristic are introduced. Passing with the equations to the phase space and marking the boundedness and asymptotical stability of the solutions in Sec. 4 the author proved, that the technical real system generated by using the "finite in time" impulse is realizing the free vibrations too. In Sec. 5 are defined the general conditions of the oscillatory property of motion according to the damping functions in the arbitrary large, bounded domain of the phase space. The analogous proof is given in the Sec. 6 in the synthetic sense. In addition some methods of synthesis are given.

27:

Skowroński, J. M., Ziemba, S. Boundedness of motion and existence and stability of limit regime in strongly non-linear, non-autonomous vibrating lumped systems. Ograniczoność ruchu, istnienie i stateczność stanu granicznego w silnie nieliniowyah, nieautonomicznych drgających układach.

Trudy Intern. Kongresa. IFAC Moskwa 1960.

Proc. Intern. Congress of IFAC Moscow 1960-edited by Butlerworths Sci. Publ. London.

In the paper is considered the dynamical system in the physical sense, the motion of which can be described in the geometrized phase space $E^{2n+1}(x_1 \dots x_{2n}, t)$ with the equations $\dot{x}_i = f_i(x_1 \dots x_{2n}, t)$ $i = 1 \dots 2n$ (a) with the adequate initial conditions. About the functions of the right sides of this equation, determined in the region Delta one: (x_i greater than -infinity and less than +infinity) it was assumed, that they satisfy all conditions for the existence and uniqueness of the solutions of (a)[M] Delta one. It was further assumed the conditions of the positive dissipation of the total energy in the system. The analysis of technical cases of "forcing" $g_i(x_1 \dots x_{2n}, t)$ is introduced.

For the systems (a) with the ultimately bounded power of the forcing source and for a general "bounded forcing case" it is proved the boundedness "in large" of the motion of investigated system.

Furthermore, assuming the same conditions, the convergence of the arbitrary solutions is shown.

Considering the limit regime "in technical sense" i.e. the closed limit domain in which are remaining the integral curves of motion with t tending to infinity (for t less than T, where T is some constant value) -it is proved the existence and asymptotic stability property of this regime.

For $q_i(x_1 \cdots x_{2n}, t)$ periodical with respect to t there is the unique periodic solubion of (a) to which tend all the other motions with t tending to infinity i.e. there is the periodic limit regime in the strict sense.

The proofs are made with the aid of showing, for the investigated system, the necessity to satisfy any sufficient conditions of T. Yoshizawa.

28:

Skowroński, J. M., Ziemba, S. The quantitative investigations of the phase-trajectories of strong non-linear mechanical systems by using the "delta" method. (In Polish).
Ilościowe badanie trajektorii fazowych silnie nieliniowych układów mechanicznych metodą "delta".
Zag.Drgan Niel. No. 3, 1961.

This paper is a continuation of the previous position (10) of this edition. There is described the delta method initiated from Jacobsen and Bonland - with generalization into several dimensional -spacious systems. This is the investigation on the projectory - planes of the 2n-dimensional, Euclidean, real phase space with the parallel way of all the projections. In many concrete examples thereaare given the applications of this method to the strong non-linear non-autonomous and autonomous systems. The general regarded form is:

$$\ddot{q}_i = f_i(q_1 \cdots q_n, \dot{q}_1 \cdots \dot{q}_n, t) \qquad i = 1 \ldots n$$

with adequate initial conditions and assumptions.

29:

Skowroński, J. M., Ziemba, S. Additional notes on the boundedness of motion (In Polish).
Pewne uzupełnienie zagadnienia ograniczoności ruchu.
Zag. Drgań Niel. No 3, 1961.

There is given a short list of the questions in the direction of the boundedness of motion by the authors data in the previous positions. There is also given a very simple proof for the boundedness of solutions for a particular case of forcing in the general moving strong non-linear system. It is proved the necessary condition of boundedness as a supplement to adequate sufficient previous conditions. There is a domain in the 2n + 1 dimensional geometrized phase-space into which all the solutions of equations of motion should be entered after the finite time.

30:

Skowroński, J. M., Ziemba, S. The estimate-criteria of the oscillatory property domain for the general dynamical systems. (In English).
Proceedings of Vibr. Problems PAN, Vol. 3, 1961.
Określenie kryteriów zakresu własności oscylacyjnych dla ogólnych układów dynamicznych.

There is considered the strong non-linear arbitrary coupled dynamical system in the phase-space, 2n-dimensional, real, Euclidean, with equations

$$\ddot{q}_i = f_i(q_1 \ldots q_n, \dot{q}_1 \ldots \dot{q}_n) \qquad i = 1 \ldots n \qquad (a)$$

with adequate initial conditions. There are assumed the conditions (necessary and sufficient) for existence, uniqueness and prolongability with $t \in \langle t_0, \infty)$, boundedness of solutions of (a) and the total asymptotical stability of the isolated singular point O(0,...,0). All the trajectories are the O^+ curves. It is introduced into the paper the original, to the technical conditions adequate, definition of the oscillatory property of motion. The motion of the system (a) is the oscillatory motion, when at least one from the coordinates q_i realises the oscillatory motion. The $q_i(t)$ realizes the oscillatory motion while there is at least one zero-point of $q_i(t)$ in the finite time i.e. for $t \in \langle t_0, T)$ where $T \in \langle t_0, \infty)$. Further it is proved that for the oscillatory property of motion it is sufficient, to find this property on one projection-plane excepted from phase-space and in the second quarter of this projection-plane.

Furthermore as the example, the concrete criteria of investigation of this property are introduced by using the generalized "delta" method, which was conceived as the qualitative space tensor by J. Skowroński.

31:

Skowroński, J. M. The oscillatory property of the motion of strong non-linear mechanical discrete systems. (In English).

Własności drgające ruchu silnie nieliniowych mechanicznych układów dyskretnych.

Arch.Mech.Stos.Vol. XII, 1961, 2.

The dynamical system

$$\ddot{q}_i = f_i(q_1 \cdots q_n \; \dot{q}_1 \cdots \dot{q}_n) \qquad i = 1 \ldots n \qquad (a)$$

with adequate initial conditions is considered in the 2n-dimensional Euclidean phase space.

There are assumed all conditions necessary and sufficient to existence, uniqueness, prolongability for $t(t_0, \infty)$, boundedness of solutions of (a) and to asymptotical stability of isolated singular point $0(0,\ldots,0)$. All the phase space trajectories are the 0^+ curves. By using some generalization of the conclusions of F. Sansone and S. Ziemba according to the plane systems (for one degree of freedom), in the paper are given the criteria of the oscillatory motion for systems described by (a). It is proved, that in the arbitrary closed domain of the phase-space the oscillatory-property-criteria are the same as in the sufficiently little surrounding region near the singular point. According to this criterion, we can regard the system (a) with arbitrary strong nonlinearity of functions f_i as the comparing linear Sansone's system. The mechanical conclusions are given.

32:

Ziemba, S. The generalized dynamical systems from the technical point of view. (In Polish).
Uogólnione układy dynamiczne z technicznego punktu widzenia.
Zag. Drgań Niel. No. 3, 1961.

In this paper the general considerations on the arbitrary physical systems with concentrated parameters are given. The author exhibits the "general dynamical systems" represented by the group of transformations $q = f(p, t)$, $t \geq t_o$, where t-parameter and p,q are the points of phase space, which synonymously describe the kinetic state of the autonomous systems of material points. These dynamical systems represent the stationary field of directions in phase space, i.e. the right sides respondible to these systems differential equations do not depend on time explicit. Then these systems do not contain the nonautonomous systems. However, if the considerations are performed in the space E_n, which in the certain general case may be treated as the phase space, the conditions of dynamical state containing more broad range of the physical systems interesting from technical point of view may be found. In the space $E_{n+1} = (x_1 \ldots x_n, t)$ with the assumption of additional condition $\frac{d}{d}\frac{t}{\tau} = 1$ both the autonomous and nonatonomous systems may be treated as the classical dynamical systems.

33:

Bogusz, W. On the limiting cycles of the self-exciting systems. (In Polish).
O dyklach granicznych ukŁadów autonomicznych. Zag.Drgań Niel. No. 3, 1961.

In this paper the author deals with a method of determination of the limiting cycles for self-exciting systems of one degree of freedom. In the first part of this paper the author gives the definitions of self-exciting vibrations and general scheme of self-exciting system and several examples with equations describing the motion under consideration. For these systems the main problem is determination of conditions of existence of the limiting cycles regarded as the periodic solution, asymptotically orbitally stable; this solution is not constant.

The author adduces several theorems allowing to resolve this problem, stating however that the determination of the region of the limiting cycles on the phase plane is practically difficult. Therefore the author used the method described by him in the paper "Determination of stability...".(A.M.S., 6, 11, 1959.) Using this method the author investigates the equation of Van der Pol and the equation of Rayleigh - the region of existence of the limiting cycles in both cases is determined. The author investigates the equation of the motion of the self-exciting systems with non-continuous characteristics by using this method.

34:

Bogusz, W. On the certain type dynamic systems. (In Polish).
O pewnym rodzaju układów dynamicznych. Zag.Drgań Niel. No. 1, 1960.

Basing on the definition of a dynamic system was proved the sufficient condition of representation of dynamic system by set of differential equations of the form

$$\dot{x}_i = f_i (x \quad \ldots \quad , t) \tag{1}$$

Assuming that functions $f_i\ (x_1, \ldots, x_n, t)$ are defined and continuous in region $-\infty < x_i < +\infty$, $-\infty < t < +\infty$ it was proved the theorem that solutions of system (1) are defined in the interval $-\infty < t < +\infty$, if a series $\sum_{v=1}^{\infty} \frac{R_v}{K_v}$ is divergent. K denotes the upper limit of the set of functions $f_i \{ f_i (x_i, \ldots x_n, t) \}$ within the circumference of radius R, round the point (0,...,0). The proof is based upongthe Schauder's theorem on the invariant point

34a:

Bogusz, W. Vibration of turbine foundations during the process of turbine starting. (In Polish).
Rosprasy Inz. 6, 2, 1958.
Drgania fundamentów turbin w procesie rozruchu.
See Appl. Mech. Rev. Vol. 12, No. 9, 1959, Rev. 4435.

35:

Bogusz, W. Determination of stability regions of dynamic non-linear systems. (In English).
Wyznaczanie obszarów stateczności układów dynamicznych nieliniowych metodą hodografu prędkości. A.S.M.S. Vol. 11, No. 6, 1959.

This paper contains the original method of qualitative analysis of non-linear system of form

$$\dot{x}_1 = f_1(x_1, x_2) \qquad \dot{x}_2 = f_2(x_1, x_2)$$

It is assumed that functions f_1 and f_2 are determined, continuous together with their partial derivatives of the first order and their Jacobian in the open set is different from zero. Using two functions F and W determining the scalar and vector product of vector $\bar{r}$ with components $(\dot{x}_1, \dot{x}_2)$ and of the vector $\bar{v}$ with components (f_1, f_2) the investigations are performed on plane $(\dot{x}_1, \dot{x}_2)$.

From the condition that these functions must be equal zero are obtained two characteristic equations for "l" and "a". The behaviour of trajectories in the phase plane depends on the signs of roots "l" and "a" and on signs of functions F and W. If the roots l are of opposite signs (two-directional field) then the singular point is saddle or focus but for roots of the same signs (one directional field) appears the stable or unstable center. All cases in plane are involved. For illustration two examples are resolved.

36:

Engel, Z. Nonlinear coupling in the centrifugal regulator. (In Polish).
Sprzeżenie nieliniowe w regulatorze odśrodkowym.
Zag.Drgań.Niel. No. 3, 1961.

The transient process of the automatic control of the rotations of engines is considered in this paper. The author investigates the mutual influence of the centrifugal regulator on the engine when the rotations are rapidly changing. It is assumed the linear damping and the linear dependency of the displacement of sleeve on the turning moment, but the supplementary moment non-linear depends on the displacement of sleeve. The investigation of Lapunov's stability problem is performed in the phase space. On the base of the shape of phase trajectories the author proves that after passing the disturbance of the regulator the transient prouess declines for nonlinear feedback with the stiff characteristic even when the damping is small.

37:

Szadkowski, J. On the boundedn [redacted] in large" of the autonomous systems. (In Polish).
O ograniczonxci "in large" układów a [redacted] nomicznych.
Zag.Drgań Niel. No. 3, 1961.

The system of equations is considered by the author

$$\frac{dX}{dt} = F(X)$$

where X and F(X) are the real matrixes representing the motion of mechanical system of n degrees of freedom in E_{2n} space and deduced from the system

$$\ddot{q}_i = F_i(q_1, \cdots q_n, \dot{q}_1, \cdots \dot{q}_n)$$

It is assumed that F(X) has the continuous partial derivatives of the first order in E_{2n} space and

$$F(0) = 0$$

without particular assumptions of F_i corresponding to the total positive energy dissipation in system (for $\delta \leq \|X\|$). On the base of the symmetrical genera[illegible] a matrix of the set of functions F(X) and on the base of properties of mechanical systems are exhibited and proved two criteria on the boundedness "in large" of motion of the considered system.

38:

Szemplinska, W. Free torsional and flexural vibrations of the cantilever beam with great flexural. (In Polish). Swobodne drgania skręcające zginające belki swpornikowej przy dużych ugięciach. Zag. Drgań Niel. No. 3, 1961.

There are considered the free vibrations of the straight slim cantilever beam as the conservative system of two degrees of freedom: torsion and flexion. The equations of the motion are linear and the natural frequencies are independent on the amplitude with assumptions of small deflexions. The assumption of great flexural deflexions has caused the necessity of concerning the displacements along the axis deformed and n of the non-deformed beam and the angle between the axis of deformed and non-deformed beam. In this the obtained equations of the motion are non-linear but the non-linear elements are small as compared with linear ones. To solve these equations the theory of quasi-linear equations (i.e. the method of small parameter) is applied. The exact equations for the beam of constant stiffness and for the relative deflexions, not exceeding 0.5 of beam length are derived.

There are given the formulae of first approximation and formulae for correcting the natural frequency. The obtained formulae allow to investigate the effect of geometrical non-linearity, which is caused by the great fluxural deflexions, on the natural frequencies of beam.

39:

Wilczkowski, J. On a certain criterion of convergency of the motions of the strongly non-linear mechanical discrete systems. (In Polish).
O pewnyn kryterium zbieznosci ruchów silnie nieliniowych, mechanicznych układów dyskretnych. Zag. Drgań. Niel. No. 3, 1961.

The paper deals with the set of the second order ordinary differential equations of type:

$$X_1 + F_i(x_1 \dots x_n, \dot{x}_1 \dots \dot{x}_n) + H_i(x_1 \cdots x_n) = G_i(x_1 \cdots x_n, \dot{x}_1 \cdots \dot{x}_n, t)$$

$i = 1, \dots, n$, describing the motion of strong non-linear non-autonomous discrete mechanical systems of "n" degrees of freedom. Transforming this set of the equations into

$$\ddot{x}_i + k_i \dot{x}_i + \omega_i^2 x_i = -f_i(x_1 \cdots x_n, \dot{x}_1 \cdots \dot{x}_n) - W_i(x_1 \cdots x_n) + G_i(x_1 \cdots x_n, \dot{x}_1 \cdots \dot{x}_n, t) \qquad i+1, \dots, n$$

that is

$$\ddot{x}_i + k_i\dot{x}_i + \omega_i^2 x_i = f_i(x_1 \ldots x_n, \dot{x}_1 \ldots \dot{x}_n, t) \quad i = 1, \ldots, n$$

the author proves that if arbitrary two solutions x and $\bar{x}_i$ starting from the arbitrary (different) initial conditions

$$x_i(t_0) = x_i^{\circ} \qquad \qquad \bar{x}_i(t_0) = x_i^{\circ}$$
$$\text{and}$$
$$\dot{x}_i(t_0) = v_i^{\circ} \qquad \qquad \dot{\bar{x}}(t_0) = \bar{v}_i^{\circ}$$

exist, if they are bounded and if

$$k_i < R_i \leqslant \frac{F_i}{x_i} \leqslant k_i - v_i \qquad i = i, \ldots, n$$

where

$$\delta F_i = \int^{t} f_i(\bar{x}_i, \ldots, \bar{x}_n, \dot{\bar{x}}_1, \ldots, \dot{\bar{x}}_n, t)dt - \int_{t_0}^{t} f_i(x_1, \ldots, x_n, \dot{x}_1 \ldots \dot{x}_n, t)dt$$
$$\delta x_i = \overset{t_0}{\bar{x}_i} - x_i$$

R_i, r_i - arbitrary positive constants,
then these solutions are asymptotically convergent.

40:

Gutowski, R., On a certain method of integration of equation of non-linear vibrationsof system with one degree of freedom. (In Polish).
O pewnej metodzie calkowania rówań drgań nieliniowych układu o jednym stopniu swobody. Biul. WAT, IX, No. 4, 1960.

In this paper a criterion of integrality of the motion equation of a material point with one degree of freedom by squareness is stated and a method af carrying out such integration, when this criterion satisfied, is given. At first the given equation is replaced by the system of equations of the first order, the right of which satisfy some conditions e.q. Cauchy-Rieman's ones. Basic conceptions of modified theory of analytic functions, corresponding to the modified Cauchy Rieman's relations, have been built. Examples of equations that can be integrated by the squareness, together with a process of integration and obtaining a solution in a finite form have been given.

41:

Lewandowski, A., An investigation of the alternating character of free non-linear vibrations by means of comparison with trajectories of linear equations. (In English). Badanie naprzemienności swobodnych tłumionych drgań nieliniowych na płaszczyźnie fazowej metodą poośnania z trajectoriami równań liniowych. Arch.Mech.Stos. 1, 1958.

Basing on the local comparison of the trajectory of the equation with trajectory of linear equation the investigations of alternativity are given. The author discusses the conditions of alternativity of motion and the way of estimation of a region of a phase plane. The author proves that if the initial conditions are represented by one point placing in the internal part of this region then the motion contains one zero place only.

42:

Kramarz, H., Ziemba, S. Influence of non-linear purely viscotic damping on the character and amplitude of forced vibrations. (In Polish).
Wpływ nieliniowego czysto wiskotycznego tłumienia na charakter i amplitudę drgań wymuszonych. Biul. WAR XIII, No. XLVI, 1959.

Results of investigations of approximative solutions of differential equation, describing forced vibrations of a set with one degree of freedom and non-linear purely viscous damping were given in this work. Resonance curves have been given and stability of that motion has been investigated.

43:

Gawroński, R. Ziemba, S. Application of the delta method in investigations of non-linear systems of electronics and automatics. (In Polish). Zastowsowanie metody delta do badania niektórych nieliniowych ukXadówelektroniki i automatyki. Biul. WAT. no. 5, 1958.

An application of the delta method to analyze non-linear systems of electronics and automatics is described. Several examples of using this method to investigate transient process in self-exciting systems such as RC generator are given and also in relaxitive systems (blocking generator and bistable multi-vibrator) and in automatic control systems. Some notes on the application and properties of delta method. are presented. This paper is the second part of the previous paper.

44:

Osiński, Z. Wilczkowski, J. <u>Application of the method of small parameter to the approximated solution of the non-linear equations differential of the aperiodic motion.</u> (In Polish);
Zastosowanie metody małego parametru do przybliżonego rozwiązania równań różniczkowych ruchu aperiodycznego.
Zag.Drgań. Niel. No. 3, 1961.

The paper concerns the application of the method of small parameter to the approximated solution of the differential equations of the non-linear (aperiodic) motion. It is considered the equation of type

$$\ddot{x} + k\dot{x} + \omega^2 x = \mu f(x, \dot{x})$$

for the case when $k > 2w$.

There is derivated the method of resolving the equations for this case and there are given the formulae for the first and second approximations. As an example of this method the equation of the motion with non-linear damping is resolved:

$$\ddot{x} + 3\dot{x} + 2x = \mu x^3$$

It is established that the time of relaxation (i.e. the time after which the amplitude is e-times smaller than the initial one) depends on the initial amplitude, if the damping is non-linear (on the contrary to the linear damping).

45:

Osiński, Z. Investigation of internal friction of metals for small frequencies using the torsional free vibrations. (In Polish

Badanie tarcia wewnętrznego w metalach [illegible] częstościach metodą swobodnych drgań [illegible]
Zag. Drgań. Niel. No. 3, 19[illegible]

To investigate the internal friction there is designed the measuring stand on which you hang up in the special handle the investigated sample which is made of wire. In this paper the author presents the results of investigations of the three twisted brass samples. There is plotted the diagram of the masimal amplitudes on dependency of time. The results of the experiments are gathered on the characteristics on which is shown the dependency of the logarithmic decrement on the amplitude (simultaneously it shows the dependency of decrement on the maximal torsional stresses) and on the frequency of vibrations.

46:

Osinski, Z. Criteria of the alternativity of the vibrating motion of the system of one degree of freedom with the non-linear elastic force and with the non-linear damling. (In Polish).

Kryteria naprzemienności ruchu drgającego układu o jednym stopniu swobody z nieliniową siłą sprężystą i z nieliniowym tłumieniem.
Zag. Drgań Niel. No. 3, 1961.

In this paper is investigated the motion of mechanical system described by the equation

$$\ddot{x} + R(\dot{x}) + S(x) = 0$$

where R (x) is the non-linear damping friction dependent on the velocity only and S(x) the non-linear elastic force. By putting on R(x) and S(x) the general conditions R(x) · x > 0 and S(x) · x > 0, in the bounded or non-bounded interval x or x, the author defines the criteria of the alternativity of motion of the system (infinite zero places) or the criteria of non-alternativity (finite zero places).
The author introduces the geometrical interpretation of the presented criteria by using the criteria "straight line" and "auxiliary straight line". For all kinds of characteristics satisfying the above condition by using these criteria may be defined the alternativity of the motion in simple and unique manner. Several examples are given.

47:
Osiński, Z. Problem of influence of the simultaneous action of the different frequency periodic exciting forces on a certain non-linear vibrating system. (In Polish). Zagadnienie wpływu jednoczesnego działania sił wymuszających okresowych o różnej częstości na pewien nieliniowy układ drgający. Zag. Drgań. Niel. No. 1, 1960.

This article deals with the influence of the simultaneous acting exciting forces of different frequencies on the vibrating system with one degree of freedom and with non-linear damping characteristics. It was provided that this characteristic may be expressed: $R(\dot{x}) = a\dot{x} + \mu x^3$ The problem is confined to the investigation of the equation of motion

$$\ddot{x} + a\dot{x} + \omega^2 x + \mu x^3 = q_1 \sin m\nu t + q_2 \sin l\nu t$$

The author resolves this equation using the modified method of a little parameter, assuming "μ" to be the little parameter. The remaining coefficients may be of arbitrarily great values. The case m = 1 and l = 2 is discussed. There is indicated the existence of dislocation of the center of vibrations when (and it takes place for this ratio only) and also the mutual influence of the amplitudes of the exciting forces on the amplitude of vibrations. At last, the author points to the possibility of designing the damping characteristics by the investigation of the motion of system influenced by two forces.

48:

Osinski, Z. Generalization of asymptotical Bogoliubov's method in the theory of the non-linear vibrations of the non-autonomous systems with heavy damping. (In Polish). Uogólnienie asymptotycznej metody Bogoliubowa w teorii nieliniowych drgań układów nieautonomicznych dla przypadku silnego tłumienia. Zag. Drgań. Niel. No. 1, 1960.

Developed by N. N. Bogoliubov, the asymptotical method for the solution of the differential equation of the vibrating motion of the non-autonomous system is applicable only inthe case of light damping, even in the case of linear equation. The author gives the generalized method permitting to resolve the equation in the case of heavy damping. The form of the investigated equation is

$$\ddot{x} + a\dot{x} + \omega^2 x = \mu F_1 (x, \dot{x}, \nu t) \tag{1}$$

We have in this equatonn an arbitrary heavy linear damping while the non-linear part of the damping appears with a little parameter. There is given also the manner of solution by this method of the equation:

$$\ddot{x} + a\dot{x} + \omega^2 x = \mu F_1 (x, \dot{x}) + F_2 (\nu t) \tag{2}$$

where the exciting force does not appear under the little parameter. For this purpose the equation (2) must by transformed to the form (1) in the way shown by the author.

49:

Osinski, Z. Influence of non-linear characteristics of internal friction damping on forced vibration. (In Polish). Wpływ nie liniowej charakterystyki tłumienia tarciem wewnętrznym na drgania symuszone. Rozpr. Inz. VICVIII, 7, 1, 25-37, 1959. See Appl. Mech. Rev. Vol. 12, No. 11, 59, Rev. 5377.

The author attempted to find the method of evaluation of non-linear damping characteristics. He has assumed that damping is non-linear function of velocity $R(v) = av + bv^3$.

Then the influence of the non-linear damping on forced vibration due to the periodic, sinusoidal force on the system with one degree of freedom with linear elastic characteristic is investigated. The differential equation of motion with approximative method being the generalization of method of small parameter is resolved.

The generalization is introduced on the assumption that the small parameter appears only in non-linear part of damping but it is not contained in linear part of damping and external force. The author has shown the influence of non-linear part of damping on the amplitude of vibrations D and dissipative energy E, and also on the average power N of acting force for case of fundamental resonance. The influence of amplitude of external force q on the rate D/q, E/q^2, N/q^2 is observed. In connection with this, the following conclusion is obtained: the non-linearity of damping characteristics may be evaluated by means of the investigations of the effect of the exciting force on the forced vibrations.

50:

Osinski, Z. Certain graphical manner of approximative solution of differential equation of forced vibrations of systems of one degree of freedom. (In Polish). Pewien sykreślny sposób przybliżonego rozwiązania różniczkowego rówania ruchu drgań symuszonych układu o jednym stopniu swobody. Zeszyty Nauk. Politechn. W-skiej, Mechanika 6, 1956.

In this paper is given the graphical method of approximations resolving an equation of forced vibrations of one degree of freedom, concepted by the present author. The form of this equation is $\ddot{x} + R(\dot{x}) + C(x) = f(t)$.

This method may be particularly applied to approximate resolving the non-linear equations. The diagrams of damping characteristics $R(\dot{x})$, elastic force $C(x)$ and external force may be obtained. Using this method both the phase trajectories on the phase plane V O X and the integral curves $x = f_1(t)$ and $v = f_2(t)$ are plotted.

51:
Osinski, Z. Forced vibrations of a system of one degree of freedom due to periodic forced, with damping characterized by a strong non-linearity. (In English). Drgania symuszone siłami odresowymi układu o jednum stopniu swobody przy dilnie nieliniosym tłumieniu. AMS Vol. XL, 1, 1959.

The author investigates the influence of non-linear damping characteristics on the motion of the system of one degree of freedom, influenced by periodic force. The differential equation is

$$\ddot{x} + R(\dot{x}) + \omega^2 x = f(t)$$

On the base of the second R, Caccioppoli theorem is proved the existence of bounded motion and asynptotically tending to periodic motion of frequency equal to frequency of external force for characteristic of damping unbounded from above. Using the integral equation the author has proved, that if certain conditions concerning initial conditions, forced amplitude and parameters of of system are satisfied, the solution of the equation is bounded. It tends asymptotically to periodic motion of frequency equal to frequency of exciting force.

Collaborating Group of Authors
of
The Department of Vibrations

1. Dr. Ing. Stefan Ziemba, ordinary professor, chief of group.
2. Dr. Ing. Janislaw Skowroński, adjunct, proxy of chief.
3. Dr. Ing. Zbigniew Osiński, adjunct.
4. Dr. Ing. Władysław Bogusz, junior lecturer.
5. Dr. Ing. Roman Solecki, junior lecturer.
6. Dr. Ing. Andrzej Lewandowski, adjunct.
7. Dr. Ing. Roman Gutowski, adjunct.
8. M. Sci. Ing. Jerzy Szadkowski, assistant.
9. M. Sci. Ing. Ryszard Miklaszewski, assistant.
10. M. Sci. Jerzy Wilczkowski, assistant.
11. M. Sci. Jan Kranz, assistant.
12. M. Sci. Bogusław Radziszewski, assistant.

POLISH ACADEMY OF SCIENCES
Institute of Basic Technical Problems

Department of Vibrations
Warsaw, Świętokrzyska 21